Firms: Profit Maximization

ECON 105: Intro to Mathematical Economics

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Old midterm question

12. The production function for good q is $q = (k + l)^{\alpha}$, where k and l are capital and labor inputs, and $\alpha > 1$. Consider the following situations:

    I The function exhibits constant returns to scale.

    II The function exhibits diminishing marginal productivities in both inputs.

    III The function has a constant rate of technical substitution.

    Which of the previous statements is correct?

    (a) I and II

    (b) I and III

    (c) III only

    (d) I, II and III

    (e) None of the options are correct

Review of the firm's problem

1. Production function $q = f(L, K)$ and input costs w and v
2. Choose inputs L and K to minimize costs $wL + vK$
   - $RTS_{l,k} = \frac{\frac{\partial f}{\partial l}}{\frac{\partial f}{\partial k}} = \frac{MPL}{MPK} = \frac{w}{v}$
3. Derive cost function $C(q)$
   - Average cost: $AC = \frac{C(q)}{q}$
   - Marginal cost: $MC = \frac{\partial C(q)}{\partial q}$
4. Choose optimal output q^* to maximize profit
   - Profit = Revenue - Cost

Profit function

$$\pi(q) = R(q) - C(q)$$

- Revenue is simply (price $*$ quantity): $R(q) = p * q$
- Cost function is what we derived from the production function

Unconstrained maximization problem $\longrightarrow$ first order condition:

FOC:

$$\frac{d\pi}{dq} = \frac{dR}{dq} - \frac{dC}{dq} = 0$$

$$\implies \frac{dR}{dq} = \frac{dC}{dq}$$

Profit function

- $\frac{dR}{dq}$ is **marginal revenue (MR)**
  - Additional revenue from producing one more good
- $\frac{dC}{dq}$ is **marginal cost (MC)**
  - Additional cost of producing one more good
- Profits are maximized when the value of producing one more good is equal to the cost
- Note: we can check the **second order condition** for a maximum point

$$\frac{d^2\pi}{dq^2} < 0$$

Short-run vs. long-run

- Unlike consumers, firms face both short-run and long-run decisions
- Economists often assume firms must pay a **fixed cost (FC)** to operate (e.g., machinery investment, contracts)
- Short-run $\longrightarrow$ pays FC no matter what
- Long-run $\longrightarrow$ firm no longer pays FC if it shuts down
- Each decision is different, but there are always 2 steps:
    1. Does the firm operate or shut down?
    2. If the firm operates, how much does it produce (q)?

Short-run problem

- Divide cost function $C(q)$ into **fixed** and **variable** costs (part that depends on q)

- Example: $C(q) = q^2 + 5q + 100$
  - $FC = 100$
  - $VC = q^2 + 5q$

- 2 choices $\longrightarrow$ 2 possible profits
  1. Shut down: $\pi = -FC$
  2. Operate: $\pi = p * q - VC - FC$

Short-run problem

- Firm will produce if:

$$p * q - VC \geq 0$$

$$p * q \geq VC$$

$$p \geq \frac{VC}{q} = AVC$$

- How much to produce? $\longrightarrow MR = MC$

E.g., $p = 10$ and $C(q) = 2q^2 + 10$

Long-run problem

- In long-run, fixed costs avoided in shut-down scenario
- 2 choices $\longrightarrow$ 2 possible profits
  1. Shut down: $\pi = 0$
  2. Operate: $\pi = p * q - TC$
- Firm will produce if:

$$p * q - TC \geq 0$$

$$p * q \geq TC$$

$$p \geq \frac{TC}{q} = ATC$$

- Produce q where $MR = MC$

Summary of shut down conditions

- Short-run: shut down if $p < AVC$
- Long-run: shut down if $p < ATC$

Shut down example

Suppose a firm has costs $C(q) = 8q^2 + 4q + 8$ and faces price p. At what price will the firm shut down in the short run? What about the long run?

Additional practice

Susan has a computer manufacturing business. Currently, she is producing only one type of computer. Her total costs are given by $C(q) = 3q^3 + 48$.

(a) Find Susan's marginal cost.

(b) Find Susan's average total cost.

(c) Find Susan's average variable cost.

(d) If the price of Susan's computers is \$9, what should she do in the short run? In the long run? What if the price is \$81? Explain.

Additional practice