# Firms: Cost Minimization

## ECON 105: Intro to Mathematical Economics

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# Old midterm question

Suppose the production function of a firm exhibits constant returns to scale. To produce 15 units of output at minimum costs, the firm uses k = 3 and l = 4. Then to produce 20 units at minimum cost the firm will use

(a) $k = 3, l = 5$

(b) $k = 4, l = 9$

(c) $k = 4, l = \frac{16}{3}$

(d) $k = 2, l = 10$

# Cost minimization problem

Choose inputs $k, l$ to produce a given quantity $q$ at the lowest cost:

$$\min_{k,l \geq 0} \quad vk + wl$$
$$\text{s.t. } f(k,l) = q$$

- Solution is the pair $k^c, l^c$, called **contingent demand** (depends on quantity $q$ the firm needs to produce)

$$k^c(w,v,q),\ l^c(w,v,q)$$

# Optimality condition

$$RTS_{l,k} = \frac{\frac{\partial f}{\partial l}}{\frac{\partial f}{\partial k}} = \frac{w}{v}$$

- **Marginal Rate of Technical Substitution** $(RTS_{l,k})$: relative productivities of labor and capital
  - Tells us the trade off capital and labor, keeping production unchanged
- **Input price ratio**: ratio of wage rate to rental rate $\frac{w}{v}$
  - Relative cost of labor compared to capital

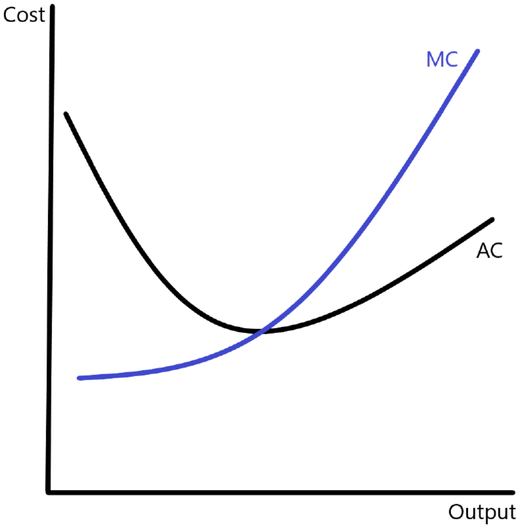
Solve for $l$ or $k$ and plug into production constraint $f(k, l) = q$

# Cost function

$$C(w, v, q) = wl^c + vk^c$$

- For a given quantity of production $q$, what are the total costs?
- Average cost per unit: $AC = \frac{C(w,v,q)}{q}$
- Marginal cost per unit: $MC = \frac{\partial C(w,v,q)}{\partial q}$
  - Cost of producing one more unit
- Both AC & MC change as production increases

# Cost curves




- AC $>$ MC
  - AC decreasing
- AC $<$ MC
  - AC increasing
- AC $=$ MC
  - AC minimized

# Cost minimization: example 1

$$f(k, l) = k^{1/2} l^{1/2}$$

# Cost minimization: example 2

$$f(k,l) = \min\{4k, 2l\}$$

# Cost minimization: example 3

$$f(k, l) = kl + 2l$$