# Elasticities

ECON 105: Intro to Mathematical Economics

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# Old midterm question

Tiny Tony has the following utility function:

$$U(x,y) = 2x^{\frac{1}{2}} + y$$

and let's assume that $\frac{I}{p_y} > \frac{p_y}{p_x}$ in order to avoid corner solutions.

1. What is the Marshallian demand for x? And for y? (5 Points)

   (a) $g_x(p_x,p_y,I) = \left(\frac{p_y}{p_x}\right)^2$ and $g_y(p_x,p_y,I) = \frac{I}{p_y} - \frac{p_y}{p_x}$

   (b) $g_x(p_x,p_y,I) = \left(\frac{p_y}{2p_x}\right)^2$ and $g_y(p_x,p_y,I) = \frac{I}{p_y} - \frac{p_y}{2p_x}$

   (c) $g_x(p_x,p_y,I) = \left(\frac{2p_y}{p_x}\right)^2$ and $g_y(p_x,p_y,I) = \frac{I}{p_y} - \frac{2p_y}{p_x}$

   (d) $g_x(p_x,p_y,I) = \left(\frac{p_y}{p_x}\right)$ and $g_y(p_x,p_y,I) = \frac{I}{p_y} - \frac{p_y}{p_x}$

# Old midterm question

2. What is the expenditure function? (5 Points)

(a) $E(p_x, p_y, \bar{u}) = p_y\left(\bar{u} - \left(\frac{p_y}{p_x}\right)^{\frac{1}{2}}\right)$

(b) $E(p_x, p_y, \bar{u}) = p_y\left(\bar{u} - \frac{p_y}{p_x}\right)$

(c) $E(p_x, p_y, \bar{u}) = p_y\left(\bar{u} - \frac{p_y^2}{p_x}\right)$

(d) $E(p_x, p_y, \bar{u}) = p_x\left(\bar{u} - \frac{p_y}{p_x}\right)$

(e) $E(p_x, p_y, \bar{u}) = p_x\left(\bar{u} - 2\frac{p_x}{p_y})\right)$

# Old midterm question

3. What is the Hicksian demand for x? And for y? (5 Points)

(a) $h_x(p_x, p_y, I) = \left(\frac{p_y}{2p_x}\right)^2$ and $h_y(p_x, p_y, I) = \bar{u} - 2\frac{p_x}{p_y}$

(b) $h_x(p_x, p_y, I) = \left(\frac{2p_y}{p_x}\right)^2$ and $h_y(p_x, p_y, I) = \frac{I}{p_y} - \frac{2p_y}{p_x}$

(c) $h_x(p_x, p_y, I) = \left(\frac{p_y}{p_x}\right)^2$ and $h_y(p_x, p_y, I) = \bar{u} - 2\frac{p_y}{p_x}$

(d) $h_x(p_x, p_y, I) = \left(\frac{p_y}{p_x}\right)$ and $h_y(p_x, p_y, I) = \frac{\bar{u}}{p_y} - 2\frac{p_y}{p_x}$

(e) $h_x(p_x, p_y, I) = \left(\frac{p_x}{p_y}\right)^2$ and $h_y(p_x, p_y, I) = \frac{\bar{u}}{p_y} - \frac{p_x}{p_y}$

# Old midterm question

4. $p_y$ increases by a small amount. Using the Slutsky equation for the good $y$, the total, substitution, and income effects are equal to: (5 Points)

   (a) $Total = -\frac{Ip_x}{p_y^2} - \frac{1}{p_x}$, $Substitution = -\frac{2}{p_x}$, $Income = -\frac{Ip_x}{p_y^2} + \frac{1}{p_x}$

   (b) $Total = -\frac{I}{p_y^2} - \frac{p_y}{p_x}$, $Substitution = -\frac{2p_y}{p_x}$, $Income = -\frac{I}{p_y^2} + \frac{p_y}{p_x}$

   (c) $Total = -\frac{I}{p_y} - \frac{1}{p_x}$, $Substitution = -\frac{2}{p_x}$, $Income = -\frac{I}{p_y} + \frac{1}{p_x}$

   (d) $Total = -\frac{I}{p_y^2} - \frac{1}{p_x}$, $Substitution = -\frac{2}{p_x}$, $Income = -\frac{I}{p_y^2} + \frac{1}{p_x}$

   (e) $Total = -\frac{I}{p_y^4} - \frac{p_y}{p_x}$, $Substitution = -\frac{2p_y}{p_x}$, $Income = -\frac{I}{p_y^4} + \frac{p_y}{p_x}$

# Important Elasticities

- Price elasticity: a 1% increase in $p_x$ changes x by $e\%$

$$e_{x,p_x} = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x}$$

- Cross-price elasticity: a 1% increase in $p_y$ changes x by $e\%$

$$e_{x,p_y} = \frac{\partial x}{\partial p_y} \cdot \frac{p_y}{x}$$

- Income elasticity: a 1% increase in $I$ changes x by $e\%$

$$e_{x,I} = \frac{\partial x}{\partial I} \cdot \frac{I}{x}$$

# Cobb-Douglas Elasticity

- Consider a consumer with a utility function $u(x, y) = xy$
- This consumer faces prices $p_x, p_y$, and has income $I$.

1. Find the marshallian demands
2. Find the hicksian demands
3. Find price and income elasticities for both marshallian and hicksian demands

# Cobb-Douglas Elasticity

# Slustky Equation and Elasticities

- A general form of the slutsky equation between two goods $i$ and $j$ is

$$\frac{\partial x_i^*(p_1, ..., p_n, I)}{\partial p_j} = \frac{\partial x_i^c}{\partial p_j} - x_j^* \frac{\partial x_i^*}{\partial I}$$

- This can be translated into an elasticity relation

$$e_{i,p_j} = e_{i,p_j}^c - s_j e_{i,I},$$

- $s_j$ is the share of income spent on good $j$, that is

$$s_j = \frac{p_j x_j^*}{I}.$$

# Gross (Marshallian) Substitutes and Complements

- Gross Substitutes: two goods $x$ and $y$ are gross substitutes if

$$\frac{\partial x^*}{\partial p_y} > 0 \text{ , or } e_{x,p_y} > 0$$

- Gross Complements: two goods $x$ and $y$ are gross complements if

$$\frac{\partial x^*}{\partial p_y} < 0 \text{ , or } e_{x,p_y} < 0$$

# Asymmetry of the gross definitions

- These definitions are not symmetric.
- If $x$ and $y$ are gross-complements, it does not mean that $y$ and $x$ are also gross-complements.
- Consider the following example (quasi-linear)

$$u = \ln(x) + y$$

- Show that $\frac{\partial x}{\partial p_y}$ and $\frac{\partial y}{\partial p_x}$ do not have the same sign.

# Net (Hicksian) Substitutes and Complements

- Net Substitutes: two goods $x$ and $y$ are said to be net substitutes if

$$\frac{\partial x^c}{\partial p_y} > 0 \text{ , or } e^c_{x,p_y} > 0$$

- Net Complements: two goods $x$ and $y$ are said to be net complements if

$$\frac{\partial x^c}{\partial p_y} < 0 \text{ , or } e^c_{x,p_y} < 0$$

# Symmetry

- If $x$ and $y$ are substitutes, they stay substitutes no matter the direction
- Definitions are symmetric:

$$\frac{\partial x^c}{\partial p_y} = \frac{\partial y^c}{\partial p_x}$$

- Show this relation holds in the quasi-linear example from earlier