Income & Substitution Effects

ECON 105: Intro to Mathematical Economics

Patrick Molligo

UCLA

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Agenda

1. Getting started with R
2. Income + substitution effects

Old midterm question

Ken wants to buy meat (m) and broccoli (b) for his dinner. His utility function is given by $u(m, b) = 2m^{\frac{1}{2}} + b^{\frac{1}{2}}$ and the prices of meat and broccoli are p_M and p_B.

1. Consider Ken's expenditure minimization problem for m and b. The tangency condition of this problem is: (4 Points)
   (a) $\left(\frac{b}{2m}\right)^{\frac{1}{2}} = \frac{p_m}{p_b}$
   (b) $\frac{1}{2}\left(\frac{m}{b}\right)^{\frac{1}{2}} = \frac{p_m}{p_b}$
   (c) $2\left(\frac{b}{m}\right)^{\frac{1}{2}} = \frac{p_m}{p_b}$
   (d) $2\frac{b}{m} = \frac{p_m}{p_b}$

Old midterm question

2. Ken's Hicksian demand for meat is: (4 Points)

   (a) $m^* = \underline{u} \left(\frac{2^2 p_b + p_m}{p_m} \right)^2$

   (b) $m^* = \underline{u}^2 \left(\frac{2 p_b}{2^2 p_b + p_m} \right)^2$

   (c) $m^* = \underline{u}^2 \left(\frac{2^2 p_b + p_m}{p_m} \right)$

   (d) $m^* = \underline{u}^{\frac{1}{2}} \left(\frac{2^2 p_b + p_m}{p_m} \right)$

Income + substitution effects

- **Substitution effect**: When p_x goes up good x now more expensive than good y; changes optimality condition:

$$MRS_{x^{*old},y^{*old}} = \frac{p_x^{old}}{p_y}$$

But new price ratio is now larger than old price ratio:

$$MRS_{x^{*old},y^{*old}} = \frac{p_x^{old}}{p_y} < \frac{p_x^{new}}{p_y}$$

  - Reduce consumption of x, increase consumption y to equalize the MRS and new price ratio.
  - Substitution effect always negative for x if p_x increases

Income + substitution effects

- **Income effect**: When p_x goes up, bundle of goods is more expensive:

$$p_x^{old} x^{*old} + p_y^{old} y^{*old} < p_x^{new} x^{*old} + p_y^{old} y^{*old}$$

- Consumer has smaller overall income; must reduce overall spending
- Income effect can be either positive or negative
  - **Normal good**: Demand increases when income increases

$$\frac{\partial x_m}{\partial I} > 0$$

  - **Inferior good**: Demand decreases when income increases

$$\frac{\partial x_m}{\partial I} < 0$$

Slutsky equation

Duality relationship between Marshallian and Hicksian demand:

$$x_h(p_x, p_y, \underline{U}) = x_m(p_x, p_y, E(p_x, p_y, \underline{U}))$$

Derivative relationship with respect to p_x:

$$\frac{\partial x_h}{\partial p_x} = \frac{\partial x_m}{\partial p_x} + \frac{\partial x_m}{\partial E}\frac{\partial E}{\partial p_x}$$

Rearrange: to find $\frac{\partial x_m}{\partial p_x}$:

$$\frac{\partial x_m}{\partial p_x} = \frac{\partial x_h}{\partial p_x} - \frac{\partial x_m}{\partial E}\frac{\partial E}{\partial p_x}$$

- Note that $\frac{\partial E}{\partial p_x} = x_h$ (Shepard's lemma)
- From duality: $x_m = x_h$ and $I = E \implies -\frac{\partial x_m}{\partial I}x_m^*$

Slutsky equation

The **Slutsky equation** splits the overall impact of a price change on demand into income and substitution effects:

$$\underbrace{\frac{\partial x_m(p_x, p_y, I)}{\partial p_x}}_{\text{Total Effect}} = \underbrace{\frac{\partial x_h(p_x, p_y, \underline{\mathsf{U}})}{\partial p_x}}_{\text{Sub. Effect}} \underbrace{- \frac{\partial x_m(p_x, p_y, I)}{\partial I} x_m}_{\text{Income Effect}}$$

Graphical interpretation

Example

Suppose $u(x,y) = x^{1/4}y^{3/4}$. Solve for Marshellian demand x_m, Hicksian demand x_h, and the indirect utility function $V(p_x, p_y, I)$. Your your solution to solve for the Slutsky equation for a change in p_x on x^* and show that it holds.

Example