Review of Concepts

ECON 105: Intro to Mathematical Economics

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Agenda

1. Concepts so far
2. Types of utility functions
3. Indirect utility

Exam Strategies

- **BRING A PENCIL**
- Come early, in-class exams can be messy
- Easy questions first
  - Move on if taking too long
  - Guess if time is up
  - Fill out scantron as you go!
- For familiar utility functions, use short-cut solutions
- Read very carefully and ask if something is unclear

Old midterm question

Suppose the utility function is given by $U(X,Y) = X^{\frac{1}{3}}Y^{\frac{1}{5}}$, which of the following statement about the utility maximization problem is correct?

a. The optimal total spending on good X is one-third of total income.

b. When the price of X increases and price of Y is unchanged, the total consumption of commodity y increases.

c. The optimal consumption level is the same if the utility function is in the form of $U(X,Y) = 5\ln(X) + 3\ln(Y)$.

d. When the price of X and Y both doubles, the optimal consumption bundle remains the same.

Things covered so far

- First Order Conditions (FOC)
  - Necessary for a max./min., but not sufficient
  - First derivative equals 0 at optimum point
- Second Order Conditions (SOC)
  - Tells us if we are at a max. or a min.
  - Second derivative $< 0 \implies$ max
  - Second derivative $> 0 \implies$ min
- Lagrange multiplier method

$$\mathcal{L} = U(x, y) + \lambda(I - p_x x - p_y y)$$

  - Take 3 partial derivatives $\implies$ FOC
  - Solve for x in terms of y
  - Plug-in solution to budget constraint
  - Solve for x^* in terms of p_x, p_y, I
  - Solve for y^* in terms of p_x, p_y, I

Things covered so far

- Utility functions - amount of utility from consuming a combo of goods

$$U(x, y)$$

- Budget constraint - amount spent on goods can't exceed income

$$p_x x + p_y y = I$$

- Marshallian demand functions - amount of each good consumed to maximize utility

$$x^*(p_x, p_y, I)$$

$$y^*(p_x, p_y, I)$$

Things covered so far

- Use MRS optimality condition to skip Lagrange

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{p_x}{p_y}$$

- Won't work if
  - MRS is constant
  - Solution leads to negative demand
- Corner solutions $\implies$ all consumption goes to one good (e.g., $x^* = 0$)

Common utility functions

- Cobb-douglas

$$U(x, y) = x^\alpha y^\beta$$

$$U(x, y) = \alpha \ln(x) + \beta \ln(y)$$

  - Consumers prefer a variety of goods
  - Log transformation preserves MRS

- Quasi-linear

$$U(x, y) = f(x) + y$$

  - E.g., $U(x, y) = \ln(x) + 3y$
  - MRS depends only on one variable

Common utility functions

- Perfect complements

$$U(x, y) = \min\{\alpha x, \beta y\}$$

  - Can't differentiate; MRS is undefined
  - Consume goods in same proportion

- Perfect substitutes

$$U(x, y) = \alpha x + \beta y$$

  - Constant MRS

Perfect complements example

$$U(x,y) = \min\{2x, 3y\}$$

Perfect substitutes example

$$U(x, y) = 4x + 3y \qquad p_x = 5,\ p_y = 3$$

Indirect utility

- Indirect utility is denoted by

$$V(p_x, p_y, I) = U\left(x^*(p_x, p_y,)I, y^*(p_x, p_y, I)\right)$$

- Level of utility (in \$) given prices and income
- Plug-in optimal x^* and y^* into original utility function
- Comparing indirect utility levels tells if if price/income changes lead to welfare gains or losses

Indirect utility example

Find the indirect utility function $V(p_x, p_x, I)$ for the following:

$$U(x, y) = \sqrt{xy}$$

Cobb–Douglas Utility

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}, \quad \alpha, \beta > 0$$

Budget constraint:

$$p_1 x_1 + p_2 x_2 = I$$

Marshallian demands:

$$x_1^* = \frac{\alpha}{\alpha + \beta} \cdot \frac{I}{p_1}, \quad x_2^* = \frac{\beta}{\alpha + \beta} \cdot \frac{I}{p_2}$$

Interpretation: Each good receives a constant expenditure share proportional to its exponent.

Perfect Substitutes

$$u(x_1, x_2) = ax_1 + bx_2, \quad a, b > 0$$

Marshallian demands:

$$(x_1^*, x_2^*) = \begin{cases} \left(\dfrac{I}{p_1}, 0\right) & \text{if } \dfrac{a}{b} > \dfrac{p_1}{p_2} \\ \left(0, \dfrac{I}{p_2}\right) & \text{if } \dfrac{a}{b} < \dfrac{p_1}{p_2} \\ \text{any } (x_1, x_2) \text{ s.t. } p_1x_1 + p_2x_2 = I & \text{if } \dfrac{a}{b} = \dfrac{p_1}{p_2} \end{cases}$$

Interpretation: Consumer buys only the cheaper good in terms of the price ratio.

Perfect Complements (Leontief)

$$u(x_1, x_2) = \min\left(ax_1, bx_2\right), \quad a, b > 0$$

Marshallian demands:

$$x_1^* = b \cdot \frac{I}{bp_1 + ap_2}, \quad x_2^* = a \cdot \frac{I}{bp_1 + ap_2}$$

Interpretation: Goods are consumed in fixed proportions

$$ax_1 = bx_2$$

Quasi–Linear Utility

$$u(x_1, x_2) = f(x_1) + bx_2$$

Marshallian demands:

$$\begin{cases} \dfrac{f'(x_1^*)}{b} = \dfrac{p_1}{p_2}, \\ x_2^* = \dfrac{I - p_1 x_1^*}{p_2} \end{cases}$$

Interpretation: The optimal x_1^* depends only on relative prices, not income. Income changes affect only x_2.