Consumer Theory, part 2

ECON 97: Economic Toolkit

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Agenda

1. MRS and optimality
2. Budget constraint graphically
3. Corner solutions

Old midterm question

An individual's utility depends on his consumption of good X and good Y. Suppose that all prices and income double. What will happen to the Marshallian demands?

(a) They will double.

(b) They will stay the same

(c) They will reduce by half

(d) There is not enough information to answer this question.

Old midterm question

If ones utility function is given by $u(x, y) = \ln(xy)$, then the person's MRS of x for y (the MRS for giving up y to get x or equivalently x is on the horizontal axis) at the point $x = 6$, $y = 15$ is

(a) 2.5

(b) 0.4

(c) 1

(d) 2

MRS optimality condition

- Notice that most Lagrangians lead to this result:

$$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{p_x}{p_y}$$

$$MRS_{x,y} = \frac{p_x}{p_y}$$

- Summarize this process into two steps
  1. **Optimality Condition**: Set $MRS_{x,y} = \frac{p_x}{p_y}$; solve for one variable
  2. **Constraint**: Plug into the budget constraint $(p_x x + p_y y = I)$ and solve for demand

Graphing the budget constraint

Why do we set MRS equal to the price ratio ($\frac{p_x}{p_y}$)?

$$\begin{aligned} p_x x + p_y y &= I \\ p_y y &= I - p_x x \\ y &= \frac{I}{p_y} - \frac{p_x}{p_y} x \end{aligned}$$

For a given set of prices & income, this is just the equation for a line[1]:

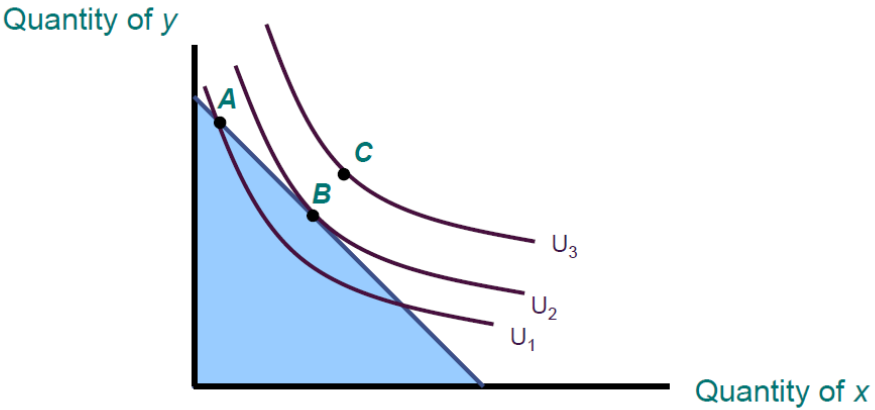
Slope: $-\dfrac{p_x}{p_y}$

Intercept: $\dfrac{I}{p_y}$

[1] In this case, the y good is on the vertical axis

Graphing the budget constraint

Tangency and the budget constraint



Issues with MRS shortcut - corner solutions

1. MRS is **constant**
   - Typically we can equate $MRS_{x,y}$ and the price ratio $\frac{p_x}{p_y}$
   - $MRS_{x,y}$ depends on one or both variables and takes range of values
   - If $MRS_{x,y}$ is a constant it might not equal the price ratio $\frac{p_x}{p_y}$
2. Negative demand
   - Our optimal demands for goods have been positive.
   - Sometimes the general solution method tells demand is **negative**.
   - Need to adjust our answer so that our demands are positive (or 0)

Constant MRS

1. **Optimality Condition**: Compare $MRS_{x,y}$ vs. $\frac{p_x}{p_y}$ to determine which variable is 0.
   - If $MRS_{x,y} > \frac{p_x}{p_y}$ then $y^* = 0$
     - Why? Value of x is larger than its cost, so we give up all of y to consume x
   - If $MRS_{x,y} < \frac{p_x}{p_y}$ then $x^* = 0$
     - Same logic as above
2. **Constraint**: Plug into the budget constraint $(p_x x + p_y y = I)$ and solve for the remaining variable

Constant MRS example

Suppose I like drinking a caffeinated beverage in the morning and only care about the amount of caffeine I get. For every cup of coffee (c) or two cups of tea (t) I drink I get 2 utility. If the price of coffee is \$4 and the price of tea is \$3 how many cups of coffee and tea should I buy?

$$u(c,t) = \alpha c + \beta t$$

Constant MRS example

Old midterm question

If David's utility function over goods X, Y is given by $U(X,Y) = 4X + 3Y$ and prices are $p_X = 5$ and $p_Y = 3$, then the optimal consumption for good X is :

(a) $\frac{I}{p_y} = \frac{I}{3}$

(b) $\frac{I}{3p_x} = \frac{I}{15}$

(c) $\frac{I}{4p_x} = \frac{I}{20}$

(d) $\frac{I}{p_x} = \frac{I}{5}$

(e) 0

Negative demand

1. **Optimality Condition**: Set $MRS_{x,y} = \frac{p_x}{p_y}$ and solve for one variable as a function of the other
2. **Constraint**: Plug into the budget constraint $(p_x x + p_y y = I)$ and solve for demand
3. **Non-negativity**: If one demand for one good is negative set that good's demand equal to zero. Return to the constraint to resolve for the remaining demand.

Negative demand example

Solve for the utility maximizing demand given $u(x, y) = \alpha \ln(x) + \beta y$. Leave prices and income as p_x, p_y, and I.

Negative demand example