Consumer Theory

ECON 105: Intro to Mathematical Economics

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Fall 2025

Agenda

1. Indifference curve review
2. Homogeneity
3. MRS and optimality

Quiz

Quiz on Thursday (10/16)

4-5 questions; 20 min.

Math basics + optimization

Old midterm question (2016)

If $U(X_1, X_2) = X_1^3 + 23X_2$, the MRS:

(a) depends on the values of X_1 and X_2

(b) depends only on the values of X_1

(c) depends only on the values of X_2

(d) is always constant

Level curves clarification

- **Marginal Rate of Substitution (MRS)** between 2 goods
  - "How much of one good am I willing to give up for more of the other good?"
  - Negative slope of indifference curve (level curve for utility function)

$$U(x_1, x_2)$$

$$\text{Slope of IC: } = \frac{dx_2}{dx_1} = -\frac{U_2}{U_1}$$

$$\text{MRS: } = -\frac{dx_2}{dx_1} = \frac{U_2}{U_1}$$

Old midterm question (2021)

If the preferences of an individual are represented by the utility function $U(X, Y)$ and the bundle of goods $A = \{X_A, Y_A\}$ lies on a higher indifference curve than $B = \{X_B, Y_B\}$, which of the following cases are possible?

(a) $X_A > X_B$ and $Y_A = Y_B$

(b) $X_A > X_B$ and $Y_A > Y_B$

(c) $X_A = X_B$ and $Y_A > Y_B$

(d) All of the above

Homogeneity of degree k

- A function is "homogeneous" of degree k if

$$f(ax, ay) = a^k f(x, y)$$

- When each argument/input is multiplied $a > 0$, the value of the function is multiplied by a^k.
- Examples:
  - $f(ax, ay) = af(x, y) \implies$ homogeneous of degree **1**
  - $f(ax, ay) = a^2 f(x, y) \implies$ homogeneous of degree **2**
  - $f(ax, ay) = f(x, y) \implies$ homogeneous of degree **0**

Old midterm question (2021)

Which of the following demand functions is **NOT** homogenous of degree zero in p_x , p_y , and I?

(a) $x = I^2/(p_x^2 + p_y)$

(b) $x = I/(p_x + p_y)$

(c) $x = I^{0.5}/(p_x^{0.5} + p_y^{0.5})$

(d) $x = I^2/(p_x^{0.5} p_y^{1.5})$

(e) $x = I(p_x^{-0.2} p_y^{-0.8})$

Homothetic preferences

- **Homothetic**: preferences don't change when income or prices are scaled by some factor
- E.g., income increases by 50%, so demand for all goods increases by 50%
- Mathematically:
    - Utility function $U(x_1, x_2)$ is homogeneous of degree 1
    - MRS $\frac{U_1}{U_2}$ is homogeneous of degree 0

Using MRS in utility maximization

- Notice that most Lagrangians lead to this result:

$$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{p_x}{p_y}$$

$$MRS_{x,y} = \frac{p_x}{p_y}$$

- Summarize this process into two steps
  1. **Optimality Condition**: Set $MRS_{x,y} = \frac{p_x}{p_y}$; solve for one variable
  2. **Constraint**: Plug into the budget constraint $(p_x x + p_y y = I)$ and solve for demand

- Solution x^*, y^* depends on prices & income and is called **Marshellian** demand

$$x^* = x_m(p_x, p_y, I),\ \ y^* = y_m(p_x, p_y, I)$$

Issues with this approach

1. MRS is **constant**
   - Typically we can equate $MRS_{x,y}$ and the price ratio $\frac{p_x}{p_y}$
   - $MRS_{x,y}$ depends on one or both variables and takes range of values
   - If $MRS_{x,y}$ is a constant it might not equal the price ratio $\frac{p_x}{p_y}$
2. Corner solutions
   - Our optimal demands for goods have been positive.
   - Sometimes the general solution method tells demand is **negative**.
   - Need to adjust our answer so that our demands are positive (or 0)

Constant MRS

1. **Optimality Condition**: Compare $MRS_{x,y}$ vs. $\frac{p_x}{p_y}$ to determine which variable is 0.
   - If $MRS_{x,y} > \frac{p_x}{p_y}$ then $y^* = 0$
     - Why? Value of x is larger than its cost, so we give up all of y to consume x
   - If $MRS_{x,y} < \frac{p_x}{p_y}$ then $x^* = 0$
     - Same logic as above
2. **Constraint**: Plug into the budget constraint $(p_x x + p_y y = I)$ and solve for the remaining variable

Constant MRS example

Suppose I like drinking a caffeinated beverage in the morning and only care about the amount of caffeine I get. For every cup of coffee (c) or two cups of tea (t) I drink I get 2 utility. If the price of coffee is \$4 and the price of tea is \$3 how many cups of coffee and tea should I buy?

$$u(c,t) = \alpha c + \beta t$$

Constant MRS example

Corner solutions

1. **Optimality Condition**: Set $MRS_{x,y} = \frac{p_x}{p_y}$ and solve for one variable as a function of the other
2. **Constraint**: Plug into the budget constraint $(p_x x + p_y y = I)$ and solve for demand
3. **Non-negativity**: If one demand for one good is negative set that good's demand equal to zero. Return to the constraint to resolve for the remaining demand.

Corner solution example

Solve for the utility maximizing demand given $u(x, y) = \ln(x) + 3y$. Prices and income are $p_x = 3, p_y = 18, I = 3$.

Corner solution example