Math Review

ECON 105: Intro to Mathematical Economics

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Agenda

1. Lagrange review
2. Practice

Probability/statistics experience

Have you taken a probability and/or statistics course?

A. Yes

B. No

Quiz: Warm-up

Consider the problem:

$$\max_{x_1,x_2} f(x_1,x_2) = x_1 + 5\ln(x_2)$$
$$s.t.\ I - x_1 - x_2 = 0,$$

How many FOCs do we need to write down?

A. 1

B. 2

C. 3

D. 4

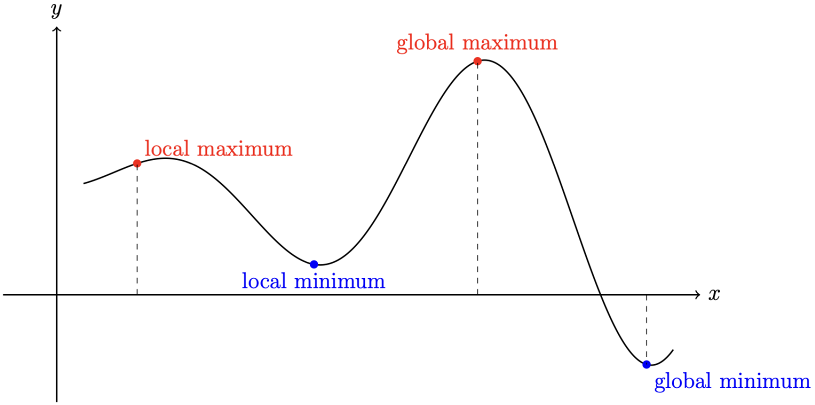
Optimization: global vs. local

- FOC and SOC tell us if we have a "local" max/min
- But it's possible there are higher/lower points
  - The absolute max/min are called "global"
  - Check the endpoints to figure out
- Consider the following:

$$f(x) = 1/3x^3 - 4x, \quad x \in [1/2, 4]$$

Optimization example, single variable

Example of local vs. global extreme values



Lagrange multiplier method

- In econ we often optimize functions subject to (s.t.) a **constraint**

$$\max_{x_1, x_2} f(x_1, x_2) \quad \text{s.t. } g(x_1, x_2) = 0$$

- Maximize the function over x_1 and x_2 given some condition for the values of x_1 and x_2

- No need to worry about SOC here, only FOC

Lagrange multiplier method

- Re-write the problem as a "Lagrangian"

$$\mathcal{L} = f(x_1, x_2) + \lambda g(x_1, x_2)$$

- The new term λ is called a Lagrange multiplier
  - Just think of λ as a third variable for now
  - $\mathcal{L}(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda g(x_1, x_2)$
- Take 3 partial derivatives and set them equal to 0, then just algebra

Lagrange example: set-up

$$\max_{x_1,x_2} f(x_1,x_2) = \frac{1}{4}\ln(x_1) + \frac{3}{4}\ln(x_2)$$

$$\text{s.t. } p_1x_1 + p_2x_2 = \mathcal{I}$$

$$\mathcal{L}(x_1,x_2,\lambda) = \frac{1}{4}\ln(x_1) + \frac{3}{4}\ln(x_2) + \lambda(\mathcal{I} - p_1x_1 - p_2x_2)$$

Lagrange example: set-up

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$$\mathcal{L}(x_1, x_2, \lambda) = \frac{1}{4}\ln(x_1) + \frac{3}{4}\ln(x_2) + \lambda(\mathcal{I} - p_1x_1 - p_2x_2)$$

FOC:

$\frac{\delta \mathcal{L}}{\delta x_1} = 0$

$\frac{\delta \mathcal{L}}{\delta x_2} = 0$

$\frac{\delta \mathcal{L}}{\delta \lambda} = 0$

Lagrange example: solution

FOC:

$\frac{\delta \mathcal{L}}{\delta x_1} = \frac{1}{4}\frac{1}{x_1} - \lambda p_1 = 0 \implies \frac{1}{4x_1} = \lambda p_1$

$\frac{\delta \mathcal{L}}{\delta x_2} = \frac{3}{4}\frac{1}{x_2} - \lambda p_2 = 0 \implies \frac{3}{4x_2} = \lambda p_2$

$\frac{\delta \mathcal{L}}{\delta \lambda} = \mathcal{I} - p_1 x_1 - p_2 x_2 = 0$

We can get rid of λ by dividing the 1st equation by the 2nd equation:

$$\frac{\frac{1}{4x_1}}{\frac{3}{4x_2}} = \frac{\lambda p_1}{\lambda p_2}$$

$$\frac{x_2}{3x_1} = \frac{p_1}{p_2}$$

$$x_2 = 3x_1 \frac{p_1}{p_2}$$

Lagrange example: solution

Finally, plug the expression for x_2 into the 3rd equation to solve for x_1^*:

$$x_2 = 3x_1 \frac{p_1}{p_2}$$

$$\begin{aligned}
\mathcal{I} &= p_1 x_1 + p_2(3x_1 \frac{p_1}{p_2}) \\
\mathcal{I} &= p_1 x_1 + (3x_1 p_1) \\
\mathcal{I} &= p_1(x_1 + 3x_1) \\
\frac{\mathcal{I}}{p_1} &= 4x_1
\end{aligned}$$

$$\frac{\mathcal{I}}{4p_1} = x_1^*$$

Lagrange example: solution

We already found that $x_2 = 3x_1\frac{p_1}{p_2}$, so just plug in x_1^*

$$x_2 = 3\left(\frac{\mathcal{I}}{4p_1}\right)\frac{p_1}{p_2}$$

$$x_2^* = \frac{3\mathcal{I}}{4p_2}$$

$$x_1^* = \frac{\mathcal{I}}{4p_1}$$

Lagrange practice # 1

$$\max_{x_1, x_2} f(x_1, x_2) = x_1^{\frac{2}{7}} x_2^{\frac{5}{7}}$$
$$\text{s.t. } p_1 x_1 + p_2 x_2 = I$$

Lagrange practice # 1

Lagrange practice # 2

$$
\begin{aligned}
\max_{x_1, x_2} u\left(x_1, x_2\right) = 2 \ln x_1 + 3 \ln x_2 \\
\text{s.t. } p_1 x_1 + p_2 x_2 = I \\
x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

Lagrange practice # 2

Lagrange practice # 3

$$\max_{x_1,x_2} f(x_1,x_2) = x_1^3 + 8x_2$$
$$s.t.\ x_1 + 2x_2 = 10$$

Lagrange practice # 3

Extra challenge: firm's optimization problem

The goal of the firm's problem is to choose the amount of production that maximizes profits:

$$\Pi(x_1, x_2) = R(x_1, x_2) - C(x_1, x_2)$$

Where $R(x_1, x_2)$ is a revenue function:

$$R(x_1, x_2) = 6x_1 + \left(\frac{99}{4}\right) x_2$$

And $C(x_1, x_2)$ is a cost function:

$$C(x_1, x_2) = \frac{x_1 x_2}{4} + \ln\left(\frac{1}{x_1^{\frac{1}{8}}} + \frac{x_2^2}{2}\right)$$

Find the pair (x_1^*, x_2^*) that maximizes profits.