Math Review

ECON 105: Intro to Mathematical Economics

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Agenda

1. Functions + Properties of Functions
2. Equations and Identities
3. Linear Functions
4. Changes and Rates of Change
5. Slope and Intercept
6. Absolute Values and Logarithms
7. Derivatives + Second Derivatives
8. Product, Quotient, and Chain Rules
9. Partial Derivatives

Warm-up

Which of the following functions is <u>not</u> differentiable?

A. $f(x) = x^3$

B. $f(x) = a^x$

C. $f(x) = |x|$

D. $f(x) = sin(x)$

Functions

- A function describes a relationship between numbers.
- Each number x has a unique number y as per the rule: $y = f(x)$.
- Examples:
  - Quadratic: $y = x^2$
  - Linear: $y = 2x$
- Independent variable: x, Dependent variable: y

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- Examples:
  - Quadratic: $y = x^2$
  - Linear: $y = 2x$
- Independent variable: x, Dependent variable: y
- The dependent variable y can be a function of one or more independent variables

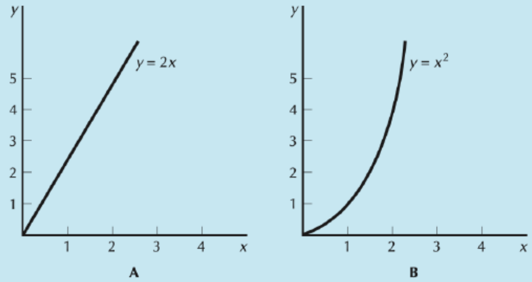
$$y = f(x_1)$$

$$y = f(x_1, x_2)$$

$$y = f(x_1, x_2, \ldots x_k)$$

Graphs

- A graph depicts the behavior of a function visually.
- Independent variable on horizontal axis, dependent variable on vertical axis.
- Sometimes in economics we do the opposite (i.e., price on y-axis, demand on x-axis)



Graphs of functions. Panel A denotes the graph of $y = 2x$, and panel B denotes the graph of $y = x^2$.

Properties of Functions

- Continuity: No jumps, can be drawn without lifting the pencil.
- Smoothness: No kinks or corners.
- Monotonicity: Function only increases or decreases.
  - Positive Monotonicity: f(x) always increases as x increases
  - Negative Monotonicity: f(x) always decreases as x increases

Equations and Identities

- An equation asks when a function is equal to a particular number, e.g.,

$$2x = 8$$
$$x^2 = 9$$
$$f(x) = 0$$

- The solution to an equation is a value of $x*$ that satisfies the equation

Equations and Identities

- An identity holds for all values of variables, e.g.,

$$(x + y)^2 \equiv x^2 + 2xy + y^2$$

- The symbol $\equiv$ means "equivalence"; the left-hand side (LHS) and the right hand-side (RHS) are equal for all ($\forall$) values of x and y
- An equation only holds for some values of the variables

Linear Functions

- A linear function has the form :

$$y = a + bx$$

  where a and b are constants.
- Example: $y = 2x + 3$
- Sometimes called an affine function

Changes and Rates of Change

- The notation Δx means "change in x".
- If x changes from x^* to x^{**} then

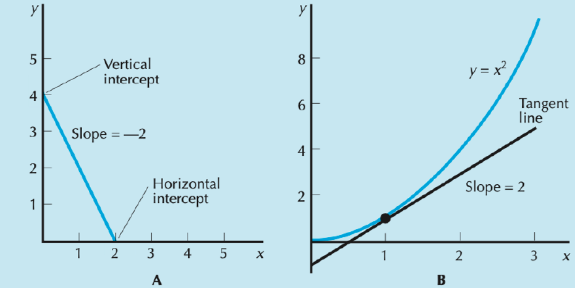
$$\Delta x = x^{**} - x^*$$

- Economics-speak: "marginal change" in x
- The rate of change is the ratio of two changes:

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Slope and Intercept

- The rate of change can be interpreted graphically as the slope.
- The vertical intercept is the value of y when $x = 0$.
- The horizontal intercept is the value of x when $y = 0$



Slopes and intercepts. Panel A depicts the function $y = -2x + 4$, and panel B depicts the function $y = x^2$.

Slope and Intercept

- A non-linear function has the property that its slope changes as x changes
- A tangent line to a function, at some point x, is a linear function with a constant slope
  - If y increases whenever x increases, then Δy will always have the same sign as Δx; slope will be positive
  - If y decreases whenever x increases, or y increases when x decreases, then Δy and Δx will have opposite signs; slope will be negative
  - Slope can change signs for certain values of x, e.g., x^2

Absolute Values

- The absolute value function $f(x) = |x|$ is defined as:

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

- Note: the absolute value is continuous but not smooth at 0

Logarithms

- The natural logarithm is used a lot in economics:

$$y = \ln(x)$$

- Special properties:
  - Defined only in the positive numbers, $x > 0$
  - $\ln(0)$ is not defined
  - $\ln(xy) = \ln(x) + \ln(y)$
  - $\ln(x^a) = a \ln(x)$
  - $\ln(e) = 1$

Derivatives

- The limit definition of a derivative of $y = f(x)$ is

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

- "The rate of change of y with respect to x for small changes in x"
- $\frac{df(x)}{dx} = f'(x)$

Common Derivatives

- If a is constant, then $\frac{da}{dx} = 0$
- If a is constant, then $\frac{d[ax]}{dx} = af'(x)$
- If a is constant, then $\frac{dx^a}{dx} = ax^{a-1}$
- $\frac{d\ln(x)}{dx} = \frac{1}{x}$
- If a is constant, then $\frac{d[a^x]}{dx} = a^x \ln(a)$
- $\frac{de^x}{dx} = e^x$
- $\frac{d[f(x) \pm g(x)]}{dx} = f'(x) \pm g'(x)$

Second Derivatives

- The second derivative is the derivative of the derivative

$$\frac{d^2 f(x)}{dx^2} = \frac{df'(x)}{dx} = f''(x)$$

- Measures curvature of a function.
  - A function with $f''(x) < 0$ is concave
  - A function with $f''(x) > 0$ is convex
  - A function with $f''(x) = 0$ is flat

The Product Rule

- If g(x) and h(x) are both functions of x define their product as

$$f(x) = g(x)h(x)$$

- Product Rule:

$$\frac{df(x)}{dx} = g(x)\frac{dh(x)}{dx} + h(x)\frac{dg(x)}{dx}$$

$$f'(x) = g(x)h'(x) + g'(x)h(x)$$

The Chain Rule

- If $y = g(x)$ and $z = h(y)$, define the composite function

$$f(x) = h(g(x))$$

- E.g., $g(x) = x^2$ and $h(y) = 2y + 3 \longrightarrow f(x) = 2x^2 + 3$
- Chain Rule:

$$\frac{df(x)}{dx} = \frac{dh(y)}{dy}\frac{dg(x)}{dx}$$

$$f'(x) = h'(y)g'(x)$$

The Quotient Rule

- Define the ratio of two functions of x as

$$f(x) = \frac{g(x)}{h(x)}$$

- Quotient Rule:

$$\frac{df(x)}{dx} = \frac{h(x)\frac{dg(x)}{dx} - g(x)\frac{dh(x)}{dx}}{h(x)^2}$$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{h(x)^2}$$

Partial Derivatives

- If y is a function of two variables x_1 and x_2 (or more)

$$y = f(x_1, x_2)$$

- Partial derivative:

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \lim_{\Delta x_1 \to 0} \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}$$

- Change in y of a marginal change in x_1, holding constant x_2

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = f_1$$

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = f_2$$

Knowledge check

Find the first derivative of the following function:

$$f(x) = \exp(x^3 + 2x)$$

Knowledge check

Find the derivative of the following function with respect to both x_1 and x_2

$$f(x_1, x_2) = \ln(x_1^{\frac{1}{3}} x_2^{\frac{2}{3}})$$